Lecture 5

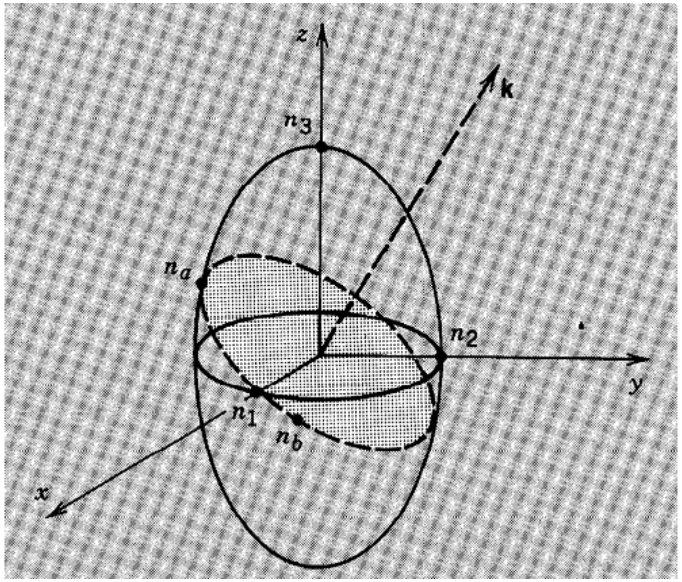
Electro-Optic Modulation

Index Ellipsoid

Consider the propagation of EM waves in the material medium. For a given direction of propagation in a crystal, in general there exist two possible (linearly polarized) modes, i.e., each mode has a defined direction of polarization (or displacement vector D is determined) and experiences a particular index of refraction.

By defining the optical axis according to the crystal orientation, we find that,

$$\frac{x^2}{n_x^{\ 2}} + \frac{y^2}{n_y^{\ 2}} + \frac{z^2}{n_z^{\ 2}} = 1 \qquad (1)$$



where x, y z, are defined as the principal axes where the electric field vector is parallel to D.

Pockel effect describes the index change in the presence of the electric field. When the index change is linearly proportional to the E field, we have the linear electro-optic effect.

It is observed that a crystal with inversion symmetry (f(x) = f(-x)) does not possess linear E-O effect.

$$\Delta n_1 = s\,E$$

$$\Delta n_2 = s\,(-E)$$

Because of the inversion symmetry, $\Delta n_1 = \Delta n_2$, or s= 0.
Instead of working directly with Δn, we could use the index ellipsoid and say that, due to the presence of the E-field vector, Eq. 1 is modified to:

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 zx + 2\left(\frac{1}{n^2}\right)_6 xy = 1 \qquad (2)$$

where x, y, z, are parallel to the principal axes of the crystal.

At E = 0,
$$\left(\frac{1}{n^2}\right)_i = \frac{1}{n_i^{\,2}} \qquad \text{(for i = 1,2,3)}$$
$$\left(\frac{1}{n^2}\right)_j = 0 \qquad \text{(for j = 4,5,6)}$$

The linear change in $(1/n^2)_i$ due to the nonzero E can be defined in term of a tensor $\{r_{ij}\}$, where:

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_{j=1}^{3} r_{ij}E_j \tag{3}$$

and $\{r_{ij}\}$ is a 3 x 6 tensor,

$$\{r\} = \begin{pmatrix} r_{11} & \cdots & r_{13} \\ \vdots & & \vdots \\ r_{61} & \cdots & r_{63} \end{pmatrix} \tag{4}$$

(Please see Yariv’s Optical Electronics for different examples of {r})

Pockels Coefficients for Some Representative Crystal Groups

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$$

Cubic $\bar{4}3m$
[e.g., GaAs, CdTe, InAs]

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$

Tetragonal $\bar{4}2m$
[e.g., KDP, ADP]

$$\begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}$$

Trigonal $3m$
[e.g., $LiNbO_3$, $LiTaO_3$]

Kerr Coefficients for an Isotropic Medium

$$\begin{bmatrix} \mathfrak{s}_{11} & \mathfrak{s}_{12} & \mathfrak{s}_{12} & 0 & 0 & 0 \\ \mathfrak{s}_{12} & \mathfrak{s}_{11} & \mathfrak{s}_{12} & 0 & 0 & 0 \\ \mathfrak{s}_{12} & \mathfrak{s}_{12} & \mathfrak{s}_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathfrak{s}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathfrak{s}_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathfrak{s}_{44} \end{bmatrix}, \qquad \mathfrak{s}_{44} = \frac{\mathfrak{s}_{11} - \mathfrak{s}_{12}}{2}$$

For the tetragonal (4$\underline{2}$m) KH_2PO_4 (KDP), the nonzero elements $r_{41} = r_{52}$, r_{63} are as follows:

	$\lambda = 0.546\ \mu m$	$\lambda = 0.633\ \mu m$
r_{41}	8.77	8
r_{63}	10.3	11
n_o	1.5079	1.502
n_e	1.4683	1.462

For this material, the crystal is uniaxial crystal as two of the indices are identical (at E=0), say $n_x = n_y$. Eq. 2 becomes:

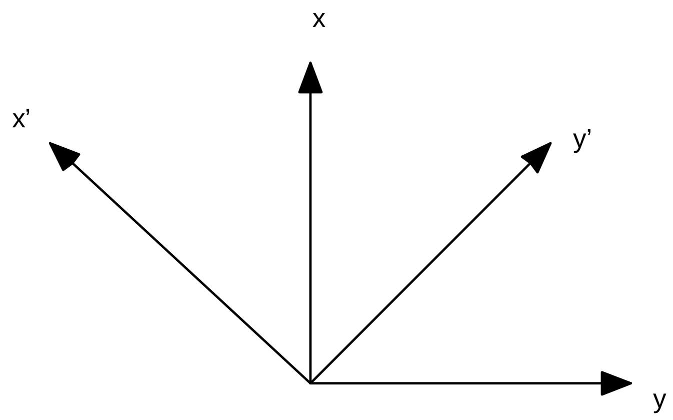
$$\left(\frac{1}{n_o^2}\right)x^2 + \left(\frac{1}{n_o^2}\right)y^2 + \left(\frac{1}{n_e^2}\right)z^2 + 2r_{41}E_x yz + 2\,r_{41}E_y zx + 2r_{63}E_z xy = 1 \qquad (6)$$

This can be interpreted by having new principal axes right after the E field is turned on!

Take the case of $E = E_z$, the index ellipsoid becomes:

$$\frac{x^2 + y^2}{n_o^{\,2}} + \frac{z^2}{n_e^{\,2}} + 2\, r_{63} E_z xy = 1 \qquad (7)$$

Define a new x'- y' axes which is obtained by a 45º counter-clockwise rotation of the x-y axes:



$$x = x' \cos 45^o + y' \sin 45^o$$
$$y = -x' \sin 45^o + y' \cos 45^o$$

Then Eq. 7 becomes:

$$\frac{\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)^2}{n_o^{\,2}} + \frac{\left(\frac{-x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)^2}{n_o^{\,2}} + \frac{z^2}{n_e^{\,2}} + 2r_{63}E_z\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)\left(\frac{-x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) = 1$$

which simplifies to:

$$\frac{x'^2 + y'^2}{n_o^2} + \frac{z^2}{n_e^2} + 2\, r_{63}E_z\left(\frac{y'^2}{2} - \frac{x'^2}{2}\right) = 1 \qquad (8)$$

or,

$$x'^2\left(\frac{1}{n_o^2} - r_{63}E_z\right) + y'^2\left(\frac{1}{n_o^2} + r_{63}E_z\right) + \frac{z^2}{n_e^2} = 1 \qquad (9)$$

With this new co-ordinate system, the new x-index is:

$$n_x'^2 = n_o^2\left(1 - n_o^2 r_{63}E_z\right)^{-1}$$

For the case that

$$r_{63}E_z \ll \frac{1}{n_o^2}$$

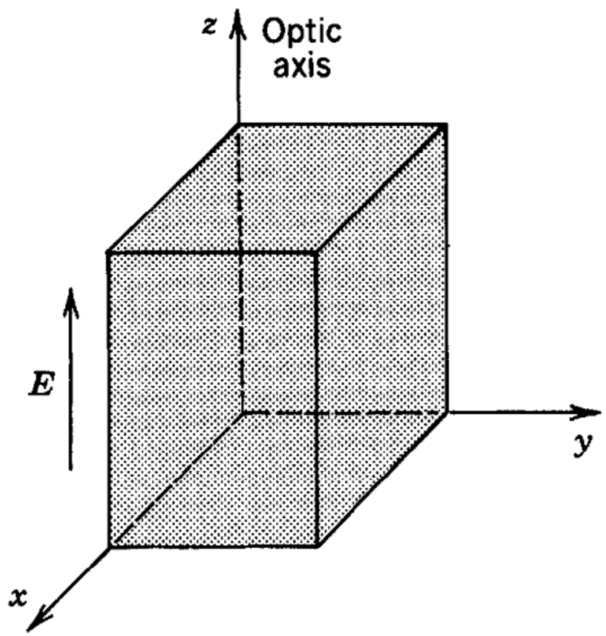
we have:

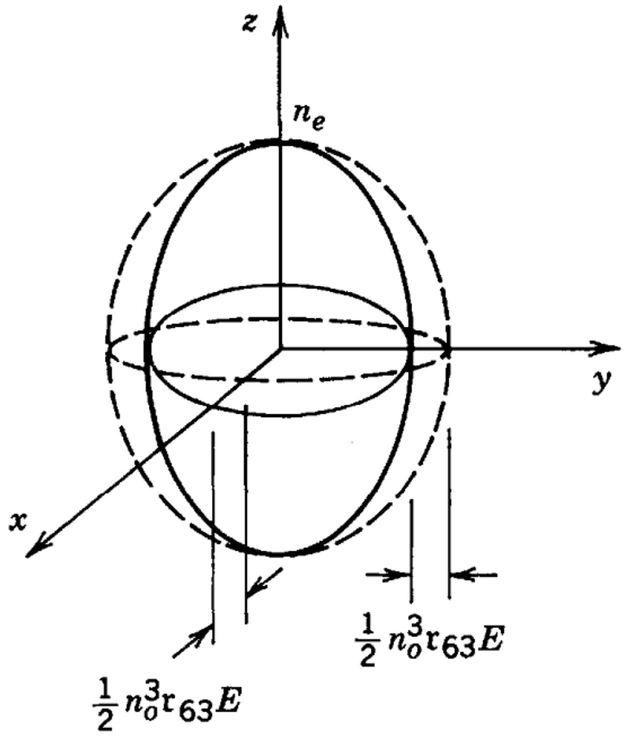
$$n_x' \approx n_o + \frac{1}{2} r_{63}E_z\, n_o^3 + \ldots \qquad (10)$$

$$n_y' \approx n_o - \frac{1}{2} r_{63}E_z\, n_o^3 + \ldots \qquad (11)$$

"push-pull"

Modification of the index ellipsoid resulting from an electric field E along the direction of the optic axis of a uniaxial tetragonal 42m crystal.





$$n_1(E) \approx n_o - \tfrac{1}{2} n_o^3 r_{63} E$$

$$n_2(E) \approx n_o + \tfrac{1}{2} n_o^3 r_{63} E$$

$$n_3(E) = n_e.$$

Thus the originally uniaxial crystal becomes biaxial when subjected to an electric field in the direction of its optic axis.

General Approach to Index Ellipsoid Equation

If we represent the terms in the equation as elements of a 3 x 3 matrix:

$$S_{11} = \left(\frac{1}{n^2}\right)_1,\ S_{22} = \left(\frac{1}{n^2}\right)_2,\ S_{33} = \left(\frac{1}{n^2}\right)_3$$

$$S_{32} = S_{23} \left(\frac{1}{n^2}\right)_4,\ \text{etc.}$$

Eq. 2 can be written in a more compact manner as

$$\sum_{i,j}^{3} S_{ij} x_i x_j = 1 \qquad (12)$$

Or in vector form as:

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} (S) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 1 \qquad (13)$$

For any point x, let's define a vector N, such that N_i is,

$$N_i = \sum_{j}^{3} S_{ij} x_j$$

Eq. 12 means N. x =1, or geometrically N is normal to the ellipsoid at x. Since for principal axes system, the principal axes are normal to the surface, so N must also be parallel to the principal axes, or we can solve for both of them by requiring:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = s \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

This is equivalent to solving:

$$\begin{pmatrix} (S_{11} - s) & S_{12} & S_{13} \\ S_{21} & (S_{22} - s) & S_{232} \\ S_{31} & S_{32} & (S_{22} - s) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \qquad (14)$$

The determinant of Eq. 14 must vanish, which gives the characteristic equation for determining the different s's.

Once s_k is determined, we can determine the radius of the corresponding principal axis.

$$|\mathrm{x}|_k = \frac{1}{\sqrt{s_k}}$$

Using this approach we can re-do the example above, the S matrix is:

$$\begin{pmatrix} \frac{1}{n_o{}^2} & r_{63}E_z & 0 \\ r_{63}E_z & \frac{1}{n_o{}^2} & 0 \\ 0 & 0 & \frac{1}{n_e{}^2} \end{pmatrix}$$

so we set:

$$\det\begin{pmatrix} \frac{1}{n_o{}^2} - s & r_{63}E_z & 0 \\ r_{63}E_z & \frac{1}{n_o{}^2} - s & 0 \\ 0 & 0 & \frac{1}{n_e{}^2} - s \end{pmatrix} = 0$$

and get:

$$\left(\frac{1}{n_e{}^2} - s\right)\left[\left(\frac{1}{n_o{}^2} - s\right)^2 - \left(r_{63}E_z\right)^2\right] = 0$$

From which we obtain:

$$s_1 = \frac{1}{n_e{}^2}$$

$$s_2 = \frac{1}{n_o{}^2} + r_{63}E_z$$

$$s_3 = \frac{1}{n_o{}^2} - r_{63}E_z$$

The principal axes can be obtained by direct substitution of s_k's back to Eq. 14.

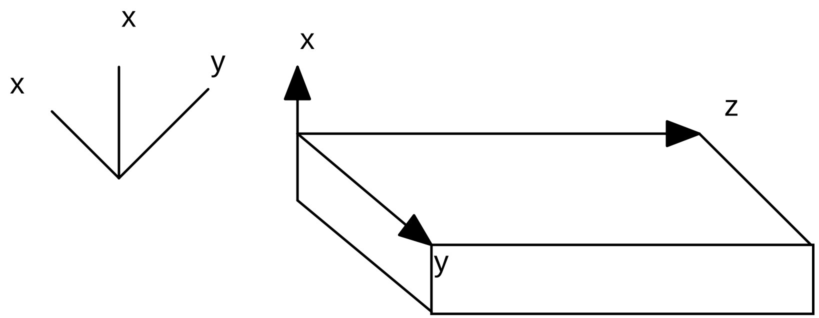
Electro-optic Retardation

We proceed by using KDP as an example, take $E=E_z$ again, and set z = 0,

from Eq. 9, we get:

$$x'^2\left(\frac{1}{n_o{}^2} - r_{63}E_z\right) + y'^2\left(\frac{1}{n_o{}^2} + r_{63}E_z\right) = 1$$

For a given cut of the crystal, the x-, y- axes are assumed to be pre-determined. With the incident light polarized along the x direction, we can resolve the electric field into x' and y' components:



$$e_{x'} = Ae^{i\left(\omega t - \frac{\omega}{c} n_{x'} z\right)} = Ae^{i\left(\omega t - \frac{\omega}{c}\left(n_o + \frac{n_o^3}{2} r_{63} E_z\right) z\right)} \qquad (15)$$

$$e_{y'} = Ae^{i\left(\omega t - \frac{\omega}{c} n_{y'} z\right)} = Ae^{i\left(\omega t - \frac{\omega}{c}\left(n_o - \frac{n_o^3}{2} r_{63} E_z\right) z\right)} \qquad (16)$$

These two components will thus travel at phase velocities (since they have different refractive index) through the medium and ends up with different phases at the output plane z = L.

The phase difference between the two components is:

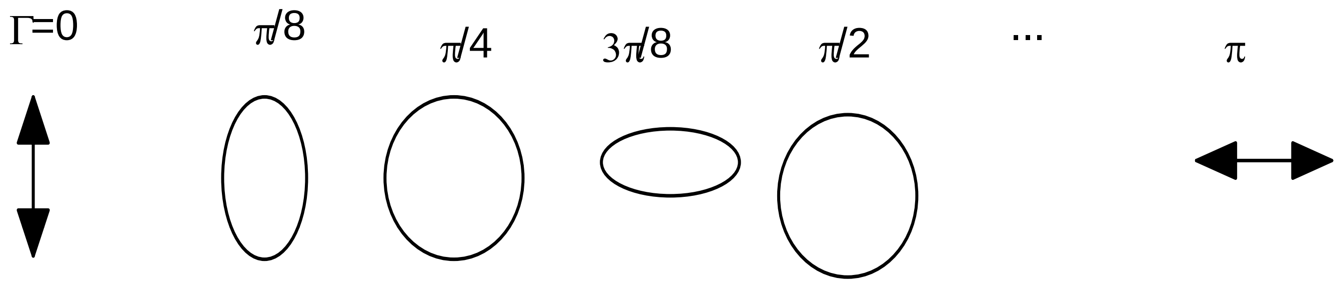
$$\Gamma = \phi_{x'} - \phi_{y'} = \frac{\omega}{c} n_o^3 r_{63} E_z L = \frac{\omega}{c} n_o^3 r_{63} V \qquad (17)$$

where V is the voltage applied across L. (Therefore the longer L is, the higher the voltage needed for a given phase change!) If Γ equals $\pi/2$, the two components at the output will combine to form a circularly polarized light. At $\Gamma = \pi$, we get the linearly polarized light again except it is 90^o from the original beam (y-polarized).

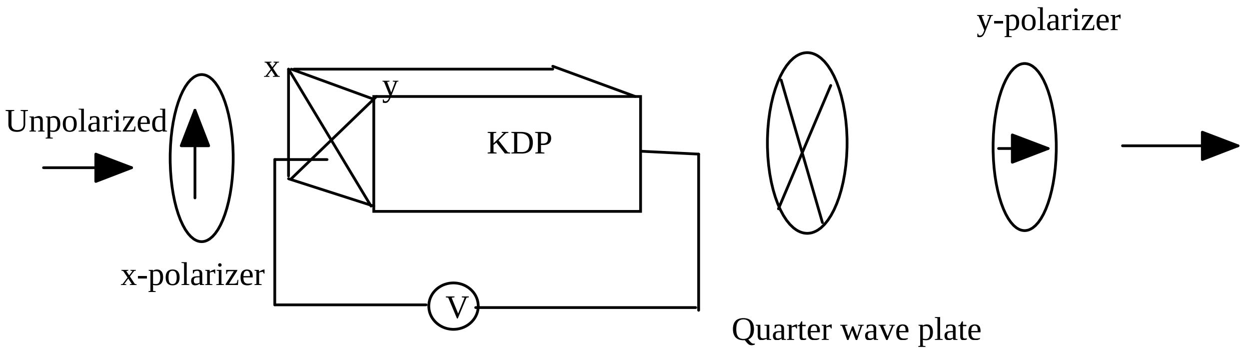
It is customary to define a half wave voltage, V_π, which is the voltage required for a π phase shift. From Eq. 17, we have

$$V_\pi = \frac{\pi c}{\omega\, n_o^3 r_{63}} \qquad (18)$$

For ADP material, for instance, r_{63} is 8.56 x 10^{-12} m/V at λ =0.5 mm, and n_o is 1.5, this gives a V_π of ~ 10 kV!



Electro-Optic Intensity Modulation



Inserting input and output polarizers as shown, we have made a simple electro-optic modulator. The modulation is based upon the projection of waves that have gone through electrically induced birefringence. The control of the intensity through a bias V is the basis of the intensity modulation of light.

After the x-polarizer and before the input plane of the E-O crystal, the x-polarized light projects equal components, with the same phase, on the x' and y' axes:

$$e_{x'}(z = 0) = A \cos \omega t = \mathrm{Re}\,(Ae^{i\omega t}) \quad (1)$$

$$e_{y'}(z = 0) = A \cos \omega t = \mathrm{Re}\,(Ae^{i\omega t}) \quad (2)$$

The intensity I is proportional to E.E*, at the input of the E-O crystal, and is equal to:

$$I(z=0) \propto \left(e_{x'} + e_{y'}\right) \cdot \left(e_{x'} + e_{y'}\right)^* = \left|e_{x'}\right|^2 + \left|e_{y'}\right|^2 = 2A^2 \qquad (3)$$

After the E-O crystal, the x' and y' components pick up additional phase $\phi_{x'}$ and $\phi_{y'}$ respectively:

$$e_{x'}(z=L) = \mathrm{Re}\,(Ae^{i(\omega t + \phi_{x'})}) \qquad (4)$$

$$e_{y'}(z=L) = \mathrm{Re}\,(Ae^{i(\omega t + \phi_{y'})}) \qquad (5)$$

where, after the quarter wave plate, an additional 90^o is added to the phase difference:

$$\Gamma = \phi_{x'} - \phi_{x'} + \frac{\pi}{2} \qquad (6)$$

The y-polarizer picks up the y-projection from both x' and y' components.

Therefore after the y-polarizer, the net electric field becomes:

$$E_{y,\,out} = (-e_{x'}\cos\frac{\pi}{4} + e_{y'}\cos\frac{\pi}{4}) = \frac{1}{\sqrt{2}}\,\mathrm{Re}\left(Ae^{i(\omega t + \phi_{x'})}(-e^{i\Gamma} + 1)\right) \qquad (7)$$

and the output intensity becomes:

$$I_{out} \propto E_{y,\,out} \cdot E^{*}_{y,\,out} = \frac{A^2}{2}(-e^{i\Gamma} + 1)(-e^{-i\Gamma} + 1) = 2\,A^2\sin^2\frac{\Gamma}{2}$$

or $$\frac{I_{out}}{I_i} = \sin^2\frac{\Gamma}{2} \qquad (8)$$

If the applied E-field is sinusoidal in time, the phase difference becomes:

$$\Gamma = \frac{\pi}{2} + \Gamma_m \sin\omega_m t \qquad (9)$$

giving the transmission as:

$$\frac{I_{out}}{I_i} = \sin^2\left(\frac{\pi}{4} + \frac{\Gamma_m}{2}\sin\omega_m t\right) = \frac{1}{2}\left(1 + \sin(\Gamma_m \sin\omega_m t)\right) \qquad (10)$$

To proceed we need to expand sin (Asinϕ) in terms of Bessel functions. This is commonly done in other branches of engineering.

$$\sin\left(\Gamma_m \sin \omega_m t\right) = 2\, J_1(\Gamma_m) \sin \omega_m t + 2\, J_3(\Gamma_m) \sin 3\omega_m t + \qquad (11)$$

Γ_m is known as the index of modulation.

For small modulation depth, $\Gamma_m << 1$, we only need to consider the first term of the expansion, also $2J_1(\Gamma_m)$ goes like Γ_m when Γ_m is small, so we have:

$$\frac{I_{out}}{I_i} \cong \frac{1}{2}\left(1 + \Gamma_m \sin \omega_m t\right) \qquad (12)$$

We can see that the intensity at the output is a replica of the index of modulation or the applied voltage.

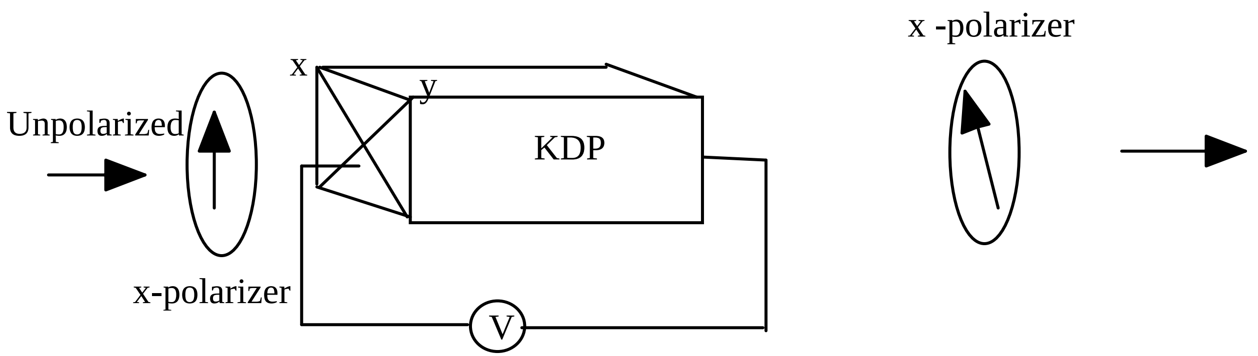
In general Γ_m may not be small, we can have higher harmonic of the signal, at $3\omega_m$, $5\,\omega_m$ and so on.

Since these harmonics are higher in frequency, they can be filtered out using a bandpass filter. However, if the input has two sinusoidal components, say ω_1 and ω_2, then Γ becomes:

$$\Gamma = \frac{\pi}{2} + \Gamma_m\left(\sin\omega_1 t + \sin\omega_2 t\right) \tag{13}$$

Following the expansion, we obtain the frequency components like $2\omega_1 - \omega_2$ or $\omega_1 - 2\omega_2$, which are close to either ω_1 or ω_2 and are thus much harder to be bandpass-filtered out. These are called intermodulation distortions.

Electro-Optic Phase Modulation



In the above scheme, if instead of the y-polarizer and the quarter wave plate, we insert a x'-polarizer that blocks the y'-component, we have effectively a phase modulator.

In this case we are interested in the phase of the x'-component as it goes through the E-O crystal:

$$e_{x'}(z = L) = \mathrm{Re}\,(Ae^{i(\omega t + \phi_{x'} + \Delta\phi_{x'})})$$

$$\phi_{x'} = -\frac{\omega}{c} n_o L$$

$$\Delta\phi_{x'} = -\frac{\omega}{c}\Delta n_{x'}\, L = -\frac{\omega n_o^{\,3}\Gamma_{63}E_z L}{2c}$$

where $E_z = E_m \sin\omega_m t$, therefore

$$e_{x'}(z = L) = A\cos\left(\omega t - \frac{\omega}{c}(n_o + \frac{n_o^3\Gamma_{63}E_m \sin\omega_m t}{2}\right)L$$
$$= A\cos(\omega t + \phi_{x'} - \delta\sin\omega_m t)$$
$$= A\cos(\omega t' - \delta\sin\omega_m t) \quad (14)$$

where δ is defined as:

$$\delta = \frac{\omega n_o^3\Gamma_{63}E_m L}{2c} = \frac{\pi n_o^3\Gamma_{63}E_m L}{\lambda} \quad (15)$$

Again we can expand the argument in the R.S. of Eq. 14 using

$$\cos(\delta\sin\omega_m t) = J_o(\delta) + 2J_2(\delta)\cos(2\omega_m t) + 2J_4(\delta)\cos(4\omega_m t) +$$
$$\sin(\delta\sin\omega_m t) = 2J_1(\delta)\sin(\omega_m t) + 2J_3(\delta)\sin(3\omega_m t) +$$

The electric field at the output becomes

$$e_{x'}(z = L) = A\left(\cos \omega t' \cos(\delta \sin\omega_m t) + \sin \omega t' \sin(\delta \sin\omega_m t)\right)$$

$$= A\left(\cos \omega t' \, J_o(\delta) + 2 \sin \omega t' \, J_1(\delta) \sin \omega_m t\right)$$

$$+ A\left(2\cos \omega t' J_2(\delta) \cos(2\omega_m t) + 2\sin \omega t' \, J_3(\delta) \sin 3\omega_m t\right) + ...$$

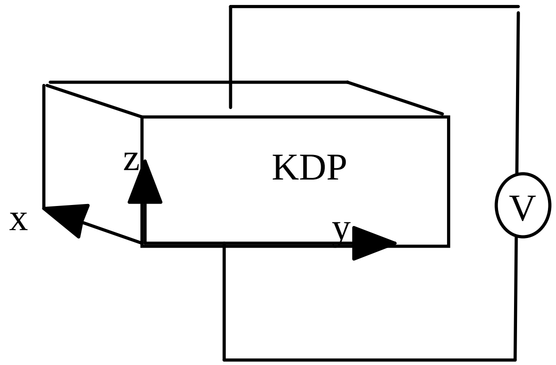
so the electric field contains many sidebands as well!

Transverse Electro-Optic Modulator

With the voltage applied along the length of the crystal, a large voltage is needed to get a sizeable electric field required for measureable E-O effect, since V = EL.

However, by suitably choosing the r_{ij}'s (or in other words, appropriately prepared the crystal cuts) so that the electric field is in the transverse direction, the voltage is not required to be large and the electric field will still be sufficient.

More importantly, the cumulative electro-optic effect can be large as the crystal can be made long. For instance, using the KDP, we can prepare the crystal as follows,



Note that E_z = V/d where d is the thickness of the region with E-field applied. When light propagates at 45^o from the z-axis along the x'-z plane, the retardation Γ becomes

$$\Gamma = \phi_{x'} - \phi_z = \frac{\omega L}{c}\left((n_o - n_e) + \frac{n_o^{\ 3}}{2}\Gamma_{63}\frac{V}{d}\right) \qquad (16)$$

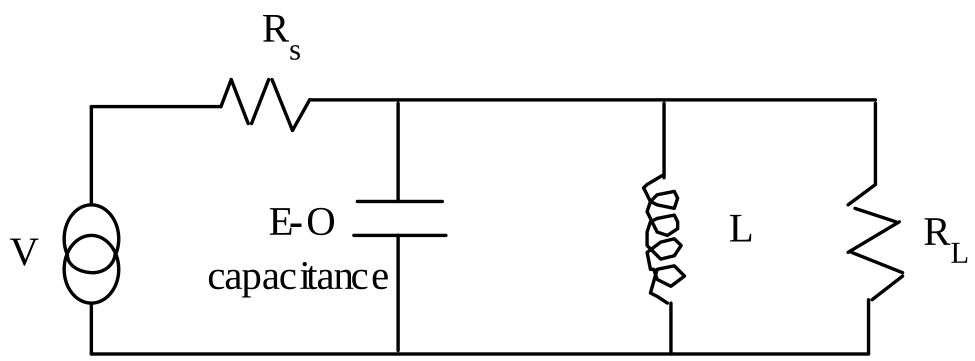
This is quite different from the previous case as Γ now depends on L/d. In the previous case, Γ (= $2\pi V n_o^3 r_{63}/\lambda$) is independent of L.

High Modulation Frequency considerations

At RF frequencies, the r's can be quite different. To drive the modulator, one has to consider the circuit element effect.

When the frequency of modulation is low, or in other words, the wavelength ($\lambda_m = c/f_m$) corresponding to the modulation frequency is large when compared to the dimension of the device, the latter can be treated as a lumped circuit element.

A simple model of the E-O modulator circuit is:



where L is for tuning the resonance frequency. A shunting resistance R_L is used so that at $\omega = \omega_o$, the impedance of the parallel RLC circuit is R_L (note that $\omega_o^2 = 1/LC$).

The value of R_L is chosen to be larger than R_s such that most of the modulation voltage appears across the E-O crystal. The bandwidth of modulation is centered at ω_o and is given as:

$$\frac{\Delta\omega}{2\pi} \cong \frac{1}{2\pi}\frac{1}{R_L C}$$

For the bulk electrode, the average drive power is:

$$P = \frac{V_m^{\ 2}}{2R_L} \qquad (17)$$

where V_m is modulation voltage, and for the KDP case discussed earlier it is given as, for a peak index of modulation of Γ_m,

$$V_m = \frac{\Gamma_m c}{\omega n_o^{\ 3} r_{63}} \qquad (18)$$

and R_L can be expressed in term of $\Delta\nu$; C is in terms of area A, separation L (remember it is the first case we discussed) and dielectric permittivity , ε.

Finally we have the drive power required for a bulk E-O modulator:

$$P = \frac{\Gamma_m^2 \lambda^2 A \varepsilon \Delta\nu}{4\pi L n_o^6 r_{63}^2} \tag{19}$$

Optical transit time effect for E-field along the direction of propagation

So far we have assumed that the electric field E is the same along the direction of light propagation, and have furthermore taken the assumption the phase difference proportional to E and L, i.e. Γ = aEL.

If the electric field changes appreciably during the optical transit time, τ_d (~ nL/c) then we need to take into account the integrated effect of the changing modulation E field:

$$\Gamma = \int_0^L a\, E(z)\, dz = a \frac{c}{n} \int_{t-\tau_d}^{t} E(t')dt' \tag{20}$$

Since E is sinusoidal in time, let's assume

$$E(t') = E_m e^{i\omega_m t'} \tag{21}$$

Then we have for Γ,

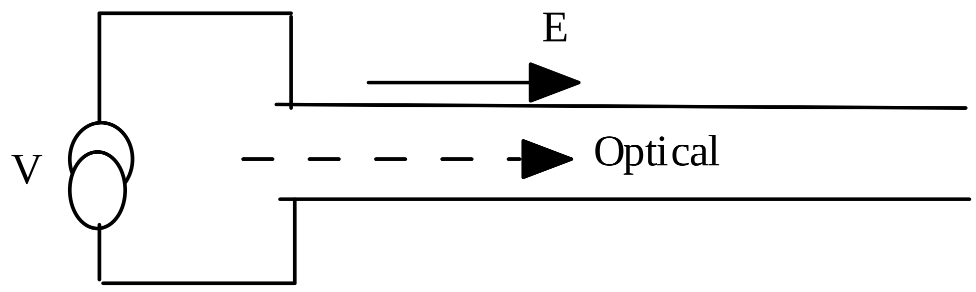
$$\Gamma(t) = a \frac{c}{n} E_m \int_{t-\tau_d}^{t} e^{i\omega_m t'} dt' = \Gamma_o \left[\frac{1 - e^{-i\omega_m \tau_d}}{i\omega_m \tau_d} \right] e^{i\omega_m t} \tag{22}$$

where $\Gamma_o = acE_m/n$. The quantity inside the square bracket in Eq. 22 is taken to be unity when the optical transit time is ignored. This corresponds to the case where $\omega_m \tau_d << 1$, or $\omega_m << \frac{1}{\tau_d} = \frac{c}{nL}$

When the optical transit time becomes noticable, for instance, when the quantity inside the square bracket is less than one, we have to evaluate the maximum modulation frequency. For example, for the [ ] = 0.9, this gives $\omega_m \tau_d = \pi/2$, or for L= 1 cm, n~ 1.5, the maximum modulation frequency is limited to c/(4nL) = 5 GHz!

Traveling-wave modulator

In the above we consider the optical transit time effect, here we consider the mismatch between the microwave and optical phase velocities. For simplicity we consider the transverse modulator where the device is long and the microwave signal travels along the electrode with the optical mode propagates in the same direction:

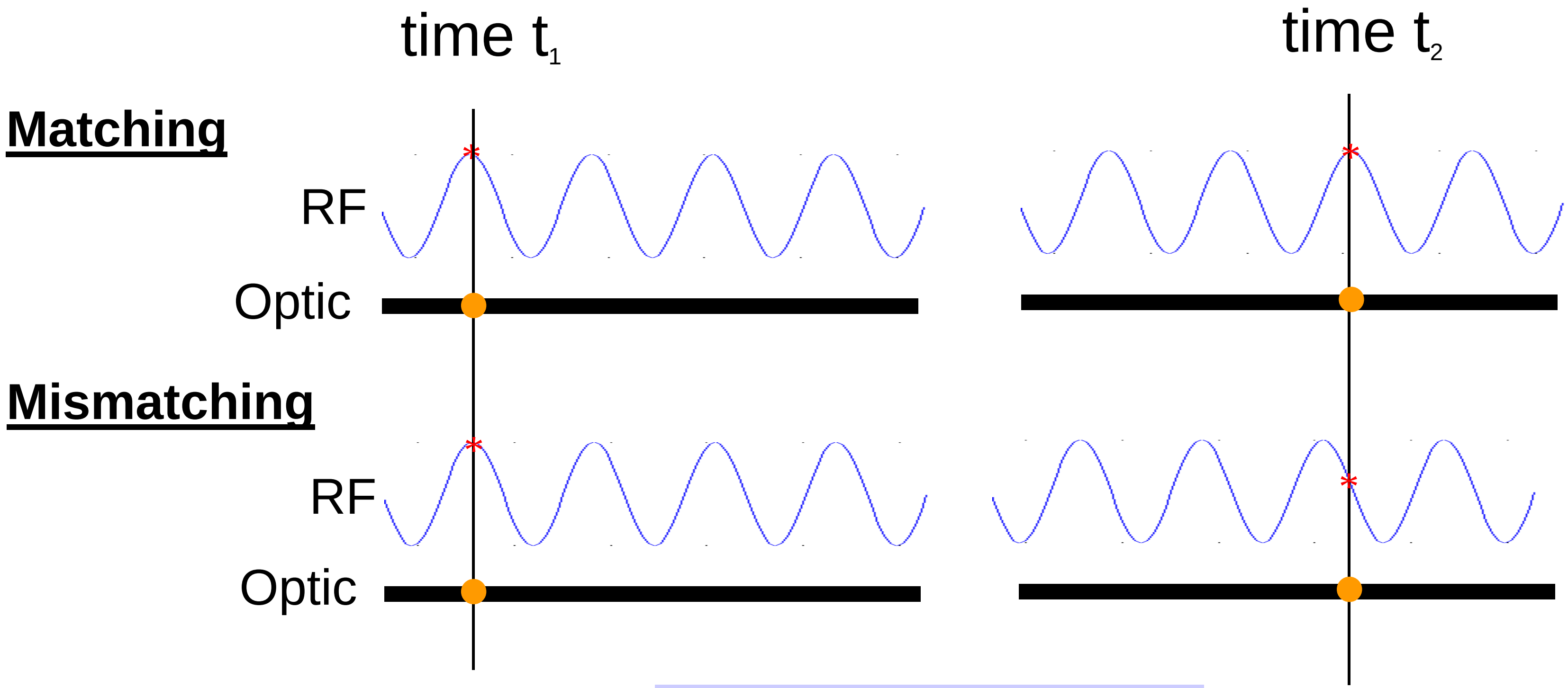


At t = t, the optical wave is at z = 0; at t = t', the wave is moved to z(t') = c/n (t'-t), where c/n is the optical phase velocity. At both instants, the optical wave "senses" different modulations, so the Γ(t) to be encountered by this element starting at t is:

$$\Gamma(t) = a\,\frac{c}{n}\int_{t}^{t+\tau_d} E(t', z(t'))\,dt' \qquad (23)$$

where E(t', z(t')) is the instantaneous modulation field at z(t') experienced by the optical phase front at t'.

Velocity Mis-match



$$\delta = (n_m - n_{geff})\omega / c$$

n_m is the microwave phase index, n_{geff} is the optical group index.

Now assume the modulation E field is a microwave signal traveling along the electrode of the modulator with phase velocity $c_m = \omega_m/k_m$:

$$E(t',z) = E_m e^{i(\omega_m t' - k_m z)} \tag{24}$$

so at t', z(t'), E is:

$$E(t',z(t')) = E_m e^{i(\omega_m t' - k_m \frac{c}{n}(t' - t))} \tag{25}$$

Substitute Eq. 25 into Eq. 23 and integrate, we get:

$$\Gamma(t) = \Gamma_o e^{i\omega_m t} \left\{ \frac{e^{i\omega_m \tau_d (1 - \frac{c}{nc_m})} - 1}{i\omega_m \tau_d (1 - \frac{c}{nc_m})} \right\} \tag{26}$$

where Γ_o is a L E_m as before. Unlike the optical-transit-time-limited case, the factor in the bracket can be made close to 1 if $1 - c/(nc_m)$ is close to zero. This is the phase velocity matching condition. Otherwise each element contributes a different $\Gamma(t)$ and reduces the net modulation effect.